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06MAT31

Third Semester B.E. Degree Examination, January 2013
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain the fourier expansion of

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases} \text{ and hence } \sum \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$ **(07 Marks)**

b. Find the half range cosine series of

$$f(x) = \begin{cases} \frac{1}{4} - x, & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{in } \frac{1}{2} < x < 1 \end{cases}$$ **(06 Marks)**

c. Obtain the constant term, first coefficients of cosine and sine terms in the fourier series expansion of the function given by the following table:

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

2 a. Find the fourier transform of

$$f(x) = \begin{cases} 1 - |x|, & \text{for } |x| \le 1 \\ 0, & \text{for } |x| > 1 \end{cases}$$

Hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$ **(07 Marks)**

b. Find f(x), if $\tau_s\{f(x)\} = \frac{s}{s^2+1}$, (fourier sine transform). **(07 Marks)**

c. Find finite fourier cosine transform of $\left(1 - \frac{x}{\pi}\right)^2$ in $0 < x < \pi$. **(06 Marks)**

3 a. Find the partial differential equation representing all planes that are at a perpendicular distance 'p' from the origin. **(07 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, for which $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0, if y is an odd multiple of $\pi/2$. **(07 Marks)**

c. Solve $y^2p - xyq = x(z - 2y)$ **(06 Marks)**

4 a. Derive arc dimensional wave equation. **(07 Marks)**

b. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the conditions u(0, y) = 0, u(π, y) = 0, u(x, ∞) = 0 and u(x, 0) = k sin2x **(07 Marks)**

4 c. Obtain the solution of the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0, t) = 0$, $u(l, t) = 0$ and $u(x, 0) = f(x)$. **(06 Marks)**

PART – B

5 a. Using the Regula-Falsi method, find the real root of the equation $xe^x - \cos x = 0$ correct to 4 places of decimals. **(07 Marks)**

b. Solve the system of equations by Gauss-Jordan method :

$x_1 + x_2 + x_3 + x_4 = 2$, $2x_1 - x_2 + 2x_3 - x_4 = -5$,
$3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$, and $x_1 - 2x_2 - 3x_3 + 2x_4 = 5$. **(07 Marks)**

c. Use the power method to find the dominant eigen value and the corresponding eigen vector correct to 2 places of decimals of the matrix

$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by taking initial vector as $[\,1\ \ 1\ \ 1\,]^T$. **(06 Marks)**

6 a. Find y when x = 0.26 using appropriate interpolation formula to the following data:

x :	0.10	0.15	0.20	0.25	0.30
Y:	0.1003	0.1511	0.2027	0.2553	0.3093

(06 Marks)

b. Fit an interpolating polynomial to the following data using Newton divided difference formula. **(07 Marks)**

x	0	1	4	8	10
f(x)	-5	-14	-125	-21	355

c. Evaluate $\int_4^{5.2} \log_e x \, dx$, using Weddle's rule by taking 7 ordinates. Hence compare with exact value. **(07 Marks)**

7 a. Obtain the Euler's equation for a variational problem in the form

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

Modify this equation when f is independent of y. **(07 Marks)**

b. Show that the equation of the curve joining the points (1, 0) and (2, 1) for which $\int_1^2 \frac{\sqrt{1 + y'^2}}{x} dx$ is an extremum is a circle. **(07 Marks)**

c. Prove that the shortest distance between two points in a plane is along the straight line joining them. **(06 Marks)**

8 a. Find the z-transform of the following:

i) $n \cos n\theta$ ii) $n \sin n\theta$ **(06 Marks)**

b. Find inverse z-transforms of the following:

i) $z \log\left(\frac{z}{z+1}\right)$ ii) $\frac{z(2z+3)}{(z+2)(z-4)}$ **(07 Marks)**

c. Solve by using Z-transform, $u_{n+2} - 4u_n = n - 1$ with $u_o = 1$ and $u_1 = 2$. **(07 Marks)**

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